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Navy Elec Lab
San Diego Calif

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20 JULY 1965

G. S. INNIS

MOST Project -3

ACTIVE SONAR TARGET CLASSIFICATION (U)

Technical Report

VOLUME II – THEORETICAL

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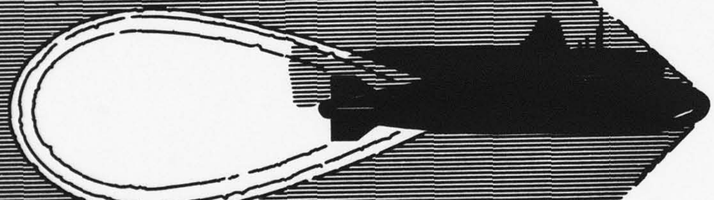

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20 July 1965

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I. INTRODUCTION

The Advance Systems Department of the Electronics Equipment Division has conducted a mathematical model of part of the ASW classification problem. This study was performed for the U. S. Navy Electronics Laboratory under the subject "Phase 2 of the ASW Classification Survey". McDonnell's management is interested in this general problem area and will support any further work that might accrue from this study. A natural extension of this study would be to perform data processing on actual data to check the feasibility of this approach.

The familiar echo ranging situation using present day sonar design was modeled as a communication problem. There are probably many theoretically possible signatures in the echo for identifying a target, but in this case the direct screw wake signature was selected. This selection was made because it seems possible to have a prior knowledge about it's shape which would permit the use of matched filter techniques. These techniques along with inter-pulse averaging are found necessary to overcome the adverse signal to noise ratio that exists. Accordingly, this study discusses the signature of the screw wake return and the approximate signal to noise ratio. Time has not permitted any detailed statistical analyses of the random noises, so the only assumption made is that they are stationary at least over some suitable period. The model given here is not necessarily accurate at this stage, but represents a foundation on which to build.

The results seem to indicate that the signal to noise power ratio of the screw echo is -41.2 db. On the other hand, data processing techniques seem to offer signal to noise improvements of 60 db. This would show that it is possible to detect these signatures.

Match filter techniques are not new and therefore by themselves do not represent the essential contribution of this proposal. The knowledge to experiment with these methods on a 7094 digital computer, and later to build the hi-speed digital circuitry for a real time sonar system is the major element.

Section II of this report discusses the spectrum of the desired signature while Section III pertains to the evaluation of the signal to noise ratio. The general theory of matched filter techniques is contained in Section IV, and Section V discusses implementation of this work for the next logical step.

II. ANALYTICAL EVALUATION OF SCREW-DOPPLER SIGNATURE FOR SQS-26 SONAR

An analytical evaluation of idealized sonar echos has been made with regard to the target identification problem. Particular attention was given to the manner in which screw-doppler information is impressed on the reflected signal. It is expected that once the approximate characteristics of this echo are known, realistic approaches to signal processing techniques can be developed to detect this signal in its true environment. Admittedly, this is a very simplified model; however, if it gives sufficient insight into the mechanism of screw-doppler, it has served its purpose.

The results seem to indicate that screw-doppler essentially produces a frequency modulation of the reflected wave. The frequency spacing of the side bands is directly related to the screw rpm although other factors must be considered. Likewise, only seven percent of the amplitude of the transmitted plane wave directly fronting on the screw is reflected with screw-doppler information. Thus, it would be expected that the predominant reflection from the hull of the submarine would almost completely mask the above effect. This would be further complicated with reverberation noise and water anomalies.

Eng.

It would be expected that matching filter techniques with signal enhancement and automatic tracking features would represent the one approach. for salvaging screw-doppler information. On the other hand, power spectral density methods would not offer these advantages. Also for the energy levels employed, it would be expected that the medium and all targets are essentially linear devices. Thus, it would not be expected to find such things as frequency biases in the echo.

In order to come to the above conclusions, a theoretical problem was modeled. It was assumed that both the transmitting vessel and target submarine were stationery in the water so that conventional doppler information was not present. Further, the transmitted pulse was of the AN/SQS-26 type, that is, a CW pulse of 0.5 seconds duration and 3500 cps carrier frequency. The screw was modeled as a four bladed type with the equivalent reflective

area located at its tip. Finally, a noiseless medium was considered with the effects of noise only considered at the end on heuristic grounds.

The transmitted or direct signal may be expressed as a sine wave that has been multiplied in the time domain by a step function in the following manner.

$$X_o(t) = (X_o \text{ Sin } 2 \pi f_o t) \quad \mu(t) \tag{1}$$

where

$$\mu(t) = 1 \qquad 0 \leq t \geq .5$$

$$\mu(t) = 0 \qquad \text{Elsewhere}$$

At this point, it is desirable to transform this CW pulse into the frequency domain and discuss the multiplicative effects that the screw tip velocity has on this spectrum. According to Lerner, the doppler effects should probably be considered as a time base change rather than a frequency shift change since the side lobe frequencies are relatively spread out relative to the center frequency. For the purpose of this report, another simplification will be made and it will be assumed that each transmitted frequency is shifted the same amount.

The intuitive reasoning for determining the shape of the frequency spectrum proceeds as follows. It was shown in equation (1) that the transmitted function is composed of the multiplication in the time domain of a sine wave and a step function. This corresponds to a special multiplication, in the frequency domain, of the two corresponding spectra called convolution. It follows that the spectrum for a sine wave is a line spectrum, in this case, at 3500 cps. and that a step function has a sine X/X distribution. Consequently, the composite spectrum is that shown in Figure 2.1 with the spectrum for an aperiodic pulse translated from a zero center frequency to a 3500 cps center frequency.

As shown in Figure 2.1, the transmitted pulse for the AN/SQS-26 sonar contains a major portion of the energy centered at 3500 cps and occupying a bandwidth of $\pm$ 2cps. The minor lobes are drawn approximately to scale. It can easily be seen that the energy in a bandwidth $\pm$ 6cps wide must be considered when working with this signal.

why?

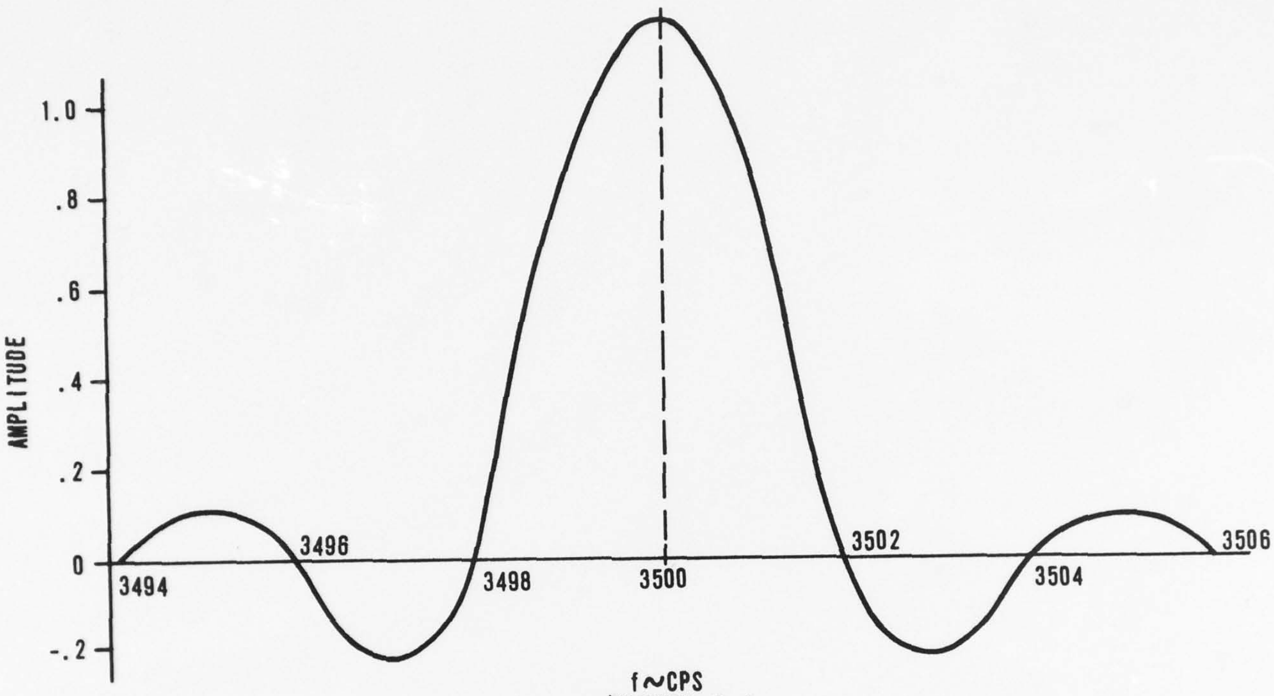

FIGURE 2.1
SIGNATURE OF DIRECT SIGNAL

Some interesting observations are that the zero crossings in the above figure are equal to $\pm$ 1/T, $\pm$ 1/2T, and so on. Thus if the pulse duration T were increased, the minor lobes would move toward the center, producing a narrower frequency spectrum. Ideally, it would be desirable to work with a discrete frequency or a line spectrum. In that case, the frequency modulation enacted by the rotating screw would produce only one infinite set of sidebands.

At this point, it is desirable to discuss the multiplicative process by which screw doppler produces sidebands in the above spectrum. For a submarine of the Nautilis class, a screw angular frequency of 10 rpm per knot will be considered for a screw with four blades. Figure 2.2 depicts a planar wave traveling with a velocity V_o feet per second and a screw with a tip velocity of $2\ \pi f_1 r$ feet per second. It is assumed here that the plane wave is moving perpendicular to the plane of the screw.

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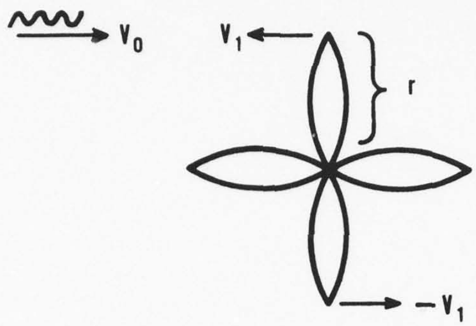

FIGURE 2.2

DIAGRAM DEPICTING SCREW MODULATING
ACOUSTICAL WAVE

Radius r of the blade is in feet and f_1 is the angular velocity in cycles per second. The tip velocity in the direction shown at the top of the figure has the effect of placing the succeeding peaks of the reflected wave closer together. This has the result that the carrier frequency of the reflected wave is shifted up in frequency by an amount:

$$2 \pi f_o \left(\frac{V_1}{V_o}\right) \qquad (2)$$

However at the same time, the negative tip velocity at the lower screw tip has the opposite result, that is, the carrier frequency of the reflected wave is shifted down in frequency. On an average basis, which is what a spectrum analysis depicts, these shifted carrier frequencies show up as side-bands about the original carrier. At the same time, the velocity components normal to the plane wave are varying in magnitude in a sinusoidal manner at the angular frequency of the screw f_1. This suggests that the reflected wave is being phase modulated.

A mathematical description for the reflected or echo signal can be written in the time domain as follows:

Linear?

$$X_E(t) = \mu(t)\, X_o \text{ Sin} \int (2\pi f_o + 2\pi f_o \frac{V_1}{V_o} \cos 2\pi f_1 t)dt \tag{3}$$

where $\mu(t)$ is again the step function described earlier. X_o is the amplitude of the plane wave which has theoretically passed through a non-attenuating medium. f_o and f_1 are respectively the frequency of the plane wave and of the screw angular velocity. A check of equation (3) shows that if $V_1 = 0$ then echo $X_E(t)$ would be of the identical form of the direct wave.

Equation (3) can be rewritten into a more convenient form by carrying out the integration, and substituting.

$$V_1 = 2\pi f_1 r$$

Further,

$$X_E(t) = \mu(t)\, X_o \text{ Sin } (2\pi f_o t + \frac{2\pi f_o r}{V_o} \sin 2\pi f_1 t)$$

$$X_E(t) = \mu(t)\, X_o \text{ Sin } (2\pi f_o t + m_f \sin 2\pi f_1 t) \tag{4}$$

Where

$$m_f = \frac{2\pi f_o r}{V_o}$$

Equation (4) is in a form readily recognizable in frequency modulation theory. Referring to the frequency domain once again, the spacing of the side bands about the carrier is entirely a function of the screw frequency f_1. The amplitudes of the side bands are a function of the deviation ratio m_f which implies that the amplitudes vary directly with water frequency f_o and inversely with water velocity V_o. The expression in equation (4) is an attempt to obtain a prior knowledge about the echo from physical reasoning.

It is interesting at this point to note the effects of a noisy medium. In this case, the water velocity V^1 can be visualized as being composed of an average velocity V_o plus a fluctuating velocity v such that v

$$V^1 = v_o + v$$

This suggests that the sidebands are fixed in frequency for a given screw frequency but vary randomly in amplitude due to water noises. This is

one of the more important conclusions and will be put to good use later on in suggesting an approach to solving the target identification problem.

It would be of interest at this point to substitute typical values into equation (4) to determine the amplitudes of the side bands for the noiseless case. Let:

$$V_o = 5 \times 10^3 \text{ feet per second}$$
$$f_o = 3.5 \times 10^3 \text{ cps}$$
$$r = 6 \text{ feet}$$

Further, if it is assumed the screw has four blades and screw is rotating at 40 rpm (4 knots) then f_1 equals 2.7 cps. Continuing in this derivation, the amplitudes can be computed from the Bessel function approximation:

$$J_n(m_f) = \sqrt{\frac{2}{\pi m_f}} \left\{ \cos \left(m_f - \tfrac{1}{2} n\pi - \frac{\pi}{2}\right) \right\} \tag{5}$$

n in equation (5) corresponds the various side bands and m_f is the previously discussed modulation index. Substituting the above values into m_f.

$$m_f = \frac{(6.28)\ (6)\ (3500)}{5{,}000} = 26.4$$

Recall that in Figure 2.1 the transmitted wave was composed of a major lobe plus many minor lobes. If the major lobe was instead a signal frequency, then the frequency modulation process would create an infinite number of frequency components f_o, $f_o \pm f_1$, $f_o \pm 2f_1$ and etc. Since in reality, the major lobe is composed of a band of frequencies, it is reasonable to expect that this same process will create bands of energy centered at each of the above frequencies.

Figure 2.3 shows curves of amplitude versus frequency for both the transmitted and received waves.

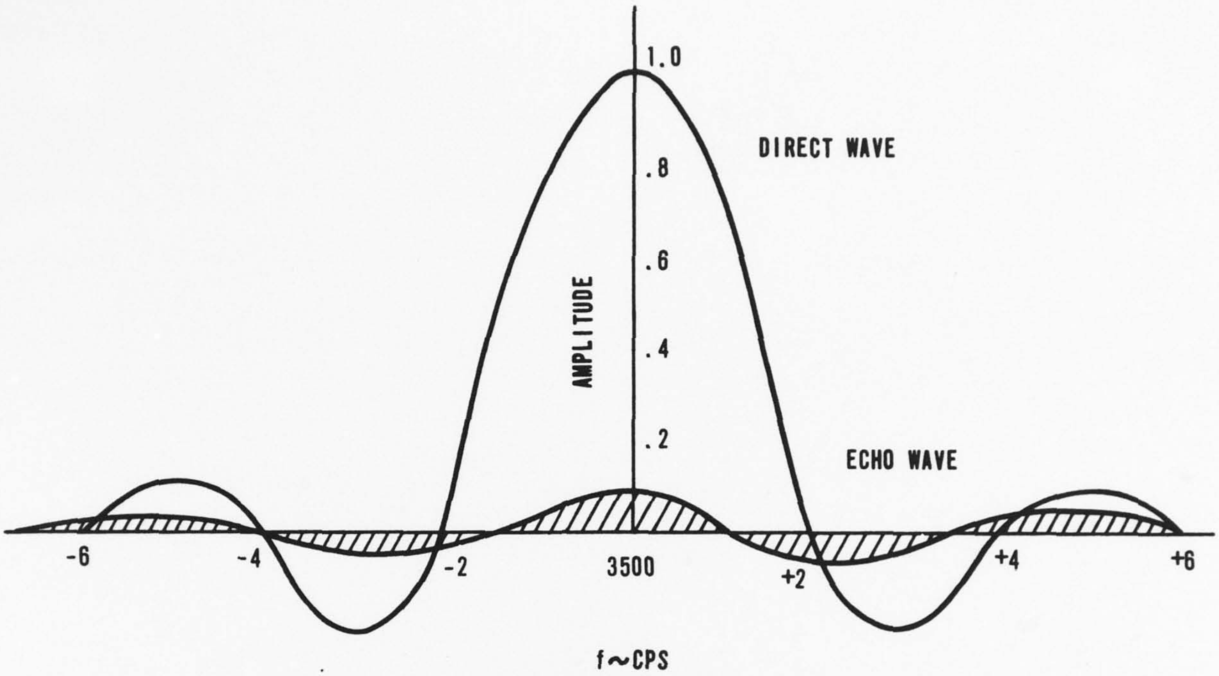

FIGURE 2.3
SIGNATURE OF DIRECT AND ECHO SIGNALS

Both curves are drawn approximately to scale. It will be recalled that in equation (5) the amplitude term is inversely related to m_f. In other words if m_f was smaller in magnitude in this example, then the received wave would be larger in magnitude relative to the transmitted wave in contrast to that shown in Figure 2.3. This seem reasonable since m_f may be visualized as a ratio of screw radius to water wave length. If r was very large relative to λ, it would be expected that there would be a canceling effect in the modulating process. On the other hand, r can not be made infinitely small to maximize the echo amplitude, since there would be very little energy delivered into the water by the screw. This indicates that r and λ must be in a reasonable relationship for equation (4) to hold.

The mathematical model indicates two additional important points that must be considered. First, the amplitude of each of the major side bands is only approximately seven percent of the amplitude of the major transmitted lobe because of the conservation of energy. Second, the cross-hatched areas in Figure 2.3 represent only the reflected wave for the major transmitted lobe. When the effects of the minor lobes are added to the reflected

wave, its spectrum becomes immeasurably more complicated.

At this point, it is appropriate to bring water attenuation, reverberation, and the swamping effect on the above signal produced by the primary reflection off of the hull of the target submarine into the picture.

There is some hope that matched filter technique may offer a solution to detecting screw-doppler. This is the subject of the next section where its signal enhancement and tracking properties are utilized. The correlative properties of these techniques may offer a means of separating screw doppler from conventional doppler which was eliminated from the mathematical model.

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III. ESTIMATION OF SIGNAL TO NOISE POWER RATIO FOR SCREW SONAR SIGNATURE

Matching filter techniques can considerably improve the signal to noise ratio of a received signal. In this case, signal to noise power improvements of 36 db are reasonable to expect. However, such drastic improvements are expensive in terms of computer time since, in the case of digital techniques, longer sample lengths necessarily denote more data points.

In order to obtain a first order approximation to the incoming signal to noise ratio, a problem has been modeled in which typical db values have been assigned to the various factors. Such factors for an echo - identification problem are acoustic source intensity, inverse square law attenuation, water attenuation, reverberation noise, and others. An exact description for the noises and wake is not known. An assumption that they are stationary and white fits the real world in most cases. In any event, a signal processing procedure based on these assumptions would be beneficial. Later these assumptions can be tightened. Results seem to indicate that the incoming signal power to prevailing noise power is approximately - 46.2 db. carry more significant digits

Further increases in the 36 db value can be obtained by using a coherent system, which, of course, does not exist in the AN/SQS-26 and -23 sonars. On the assumption that an equal amount of information is contained in the phase information as is contained in the amplitude information, an additional 36 db could be derived in this manner. It would be more reasonable to expect that amplitude and phase fluctuations are not 100 percent statistically independent so that a more realistic gain would be 20 db. Signal stacking (Ensemble Averaging) is another method by which improvements can be realized. Stacking in time has to do with two dimensional cross-correlations first across the signal and then averaging across pulses. Likewise, spatial stacking pertains to averaging signals that have traversed different transmission paths. Again, unless the paths are widely separated, they are not statistically independent, and consequently only a 20 db increase can be expected.

Through the use of at least one or more of the above methods, it seems possible to detect a screw-doppler signal. The analysis of the methods of

matched filters will be delayed until a later section. However, the method by which the - 46.2 db incoming signal to noise ratio was derived is explained below.

For the echo-identification problem the level of noise referenced to one micro-bar was estimated; likewise, the amplitude strength of the desired signal due to FM modulation was estimated. In effect, the strength of the noise was much larger than the strength of the desired signal, resulting in the - 46.2 db ratio. A large value of noise is to be expected because of larger reverberation components and strong returns from the main hull which, in this case, is considered to be noise.

A problem was modeled in which the target submarine was at a range of 10 miles or approximately 20,000 yards. The beam width of the acoustical energy radiated by the transmitting ship was selected to be 45 degrees in azimuth and 45 degrees in elevation. A bottom bounce situation was considered to exist, in which case, because of the expected beam refracting and channeling effects, the area irradiated at maximum range was considered to be 16,000 yards in azimuth and 10,000 yards in elevation. The elevation is taken to be the beam width normal to the propagation axis.

According to Reference (2), the secondary flow of sound intensity I_s can be expressed as

$$I_s = \frac{I_i \, \sigma\alpha}{4\pi \, 4r^4} \qquad (1)$$

or,

$$I_s(db) = I_i(db) + \sigma \ (db) + \alpha \ (db) - r^4 \ (db) \qquad (2)$$

I_i is the source sound intensity and is taken here to be equal to + 136 db above one microbar. This is the equivalent intensity at the target, taking into account the directivity of the beam. σ is the effective area of the target, taking into account the area reflection coefficient, cross sectional area, and direction of reflected sound. Thus

$$\sigma = \mu\sigma'\beta$$

where μ is the reflection coefficient, σ' is the cross sectional area and β is direction of the reflected sound. The rotating screw and the

associated turbulent wake behind it, still rotating with a frequency f_1, was assigned an area of 100 square yards. Thus, the wake intercepts only 6×10^{-6} of the radiated energy or a loss of -62 db takes place. The turbulent wake is expected to be far different from a perfect reflecting surface. Consequently, μ is assigned a loss factor of -6 db. The reflected sound is assumed to occupy a semisphere so β = 3 db. Substituting into equation (3) to find the screw wake effective area,

$$\alpha_s = (-62) + (-6) + (3) = -65\text{db} \qquad (4)$$

Regarding α_s, the dimensions of the wake were considered large ralative to the wave length of the sound transmission, so that the theory of reflections from a large surface applies.

As a sound radiates outward from a source, its intensity diminishes in proportion to the square of the distance from the source. The reflected or secondary flow from the target back to the original source also decays in proportion to the square of the distance, in this case, from the target. Both these phenomena can be incorporated into one expression such as equation (1) where the secondary flow is shown to vary inversely with the fourth power of the distance r. Substituting in the numbers, 20,000 yards raised to the fourth power equals approximately 1.6×10^{17}. Thus, the factor r^4 in equation (2) equals - 172 db.

Finally, the attenuation factor α in equation (3) must be considered. The attenuation of sound intensity as it propagates outwardly is composed of two parts: the losses due to the conversion of sound energy to heat, and the losses due to the scattering effects caused by nonhomogeneities in the water. The small nonhomogeneities cause refractions to take place, in which case, the new wave fronts interact with the original wave, dissipating energy. A value for α of - 6 db was obtained from standard attenuation chart for frequency range of 0-4 kcs and a two-way distance of 40,000 yards. I_s (db), the intensity of the screw secondary flow can now be computed by substituting these numbers into equation (2).

$$I_s = 136 + (-65) + (-6) + (-172) = -107 \text{ db} \qquad (5)$$

A comparison of the signal to noise power in the receiver is the main objective of this section. The strength of the signal energy E_s that is obtainable from the secondary acoustic flow is derived from

$$E_s = I_s g \qquad (6)$$

where g is the receiver gain. For an antenna with a 15 degree beam, g computes to be about 28 db. Thus,

$$E_s = -107 + 28 = -79 \text{ db} \qquad (7)$$

The value in equation (7) must now be set aside while strength of the noise is computed. The noise for this model is composed primarily of sea noise, reverberation, and the large reflected signal from the submarine hull. Since each of these noises can exist independent of the others, it is reasonable to expect that these quantities are additive, thus the db level for the total noise can be derived from the relationship:

$$N_T(\text{db}) = 10 \log \left(\log^{-1} \frac{N_S}{10} + \log^{-1} \frac{N_R}{10} + \log^{-1} \frac{N_H}{10}\right) \qquad (8)$$

N_s is the sea noise in db, N_R is the reverberation noise, and N_H is the main hull reflection noise. In conventional echo-ranging, N_H is the desired signal, but in the case where screw doppler is to be detected it becomes a noise.

A value for N_s was obtained from standard curves for a sea state 3 and 0 to 4 kcs frequency range and was found to be approximately -45 db.

The reverberation noise, at a time that corresponds to the time it takes for a signal to travel 20,000 yards and back, is quite small. Therefore, a value for N_R of -33 db was arbitrarily chosen.

N_H is an echo return similar to the screw doppler return. Thus, all the terms in equations (1) and (6) remain the same, as in the previous, case except for the target effective area.

$$\sigma = \mu\sigma'\beta \qquad (9)$$

For the case of the hull reflection, the reflection coefficient is assumed to be 0.6, σ', the cross sectional area is taken as 3,000 square yards, and β, the direction of the reflected sound is again assumed to be 3 db,that is, all the energy is reflected in a hemisphere. A ratio of the new and old values σ^* gives:

$$\sigma^* = 10 \log \left(\frac{.6 \times 3000 \times 2}{.25 \times 100 \times 2}\right) = 18.6 \text{ db} \qquad (10)$$

Referring back to equation (7), E_s was found equal to -79 db. Common reasoning would dictate that N_H would be a stronger return by the factor of 18.6 db in equation (9). Therefore,

$$N_H = -79 + 18.6 = -60.4 \text{ db} \qquad (11)$$

All the terms are now available for inserting in equation (8) for calculating N_T.

$$N_T = 10 \log \left(\log^{-1} \frac{-45}{10} + \log^{-1} \frac{-33}{10} + \log^{-1} \frac{-60.4}{10}\right) = 32.8\text{db} \qquad (12)$$

The -32.8 db obtained is only slightly more positive than the -33db for the reverberation noise. Since N_T is strongly dependent on the estimate for the reverberation noise, it would be worth while to try and obtain a more accurate estimate. Note also that for the problem as modeled and no reverberation noise the conventional return has a -15.4 db signal to noise ratio. With filtering, to be discussed later, this value is raised to + 6 db which is fairly realistic.

The final objective of this section can now be realized, that of determining the signal to noise power ratio.

$$\rho = \frac{E}{N_T} = -79 -(-32.8) = -46.2\text{db} \qquad (13)$$

Clearly, signal enhancement techniques are necessary if there is to be any chance of recovering the signal under these adverse conditions. The first section gave the screw doppler signature or the distribution of acoustic pressure with frequency. Likewise, this section derived the total energy in the signature as well as in the noise. From this information and on the assumption that the noise is uniformily distributed in the frequency range 0 to 4 kcs, the relative energies can be depicted as areas as shown in Figure 3.1. The large cross hatched area represents the noise and the small spot located at 3500 $\pm$ 6 cps represents the desired signal. Obviously, a simple band pass filter centered at 3500 cps would improve the signal to noise ratio ρ tremendously. In fact, this operation alone can increase ρ to -21 db. The object of the next section is to show how matched filters or comb filters can further reduce the filtered spectrum and thereby further increase ρ.

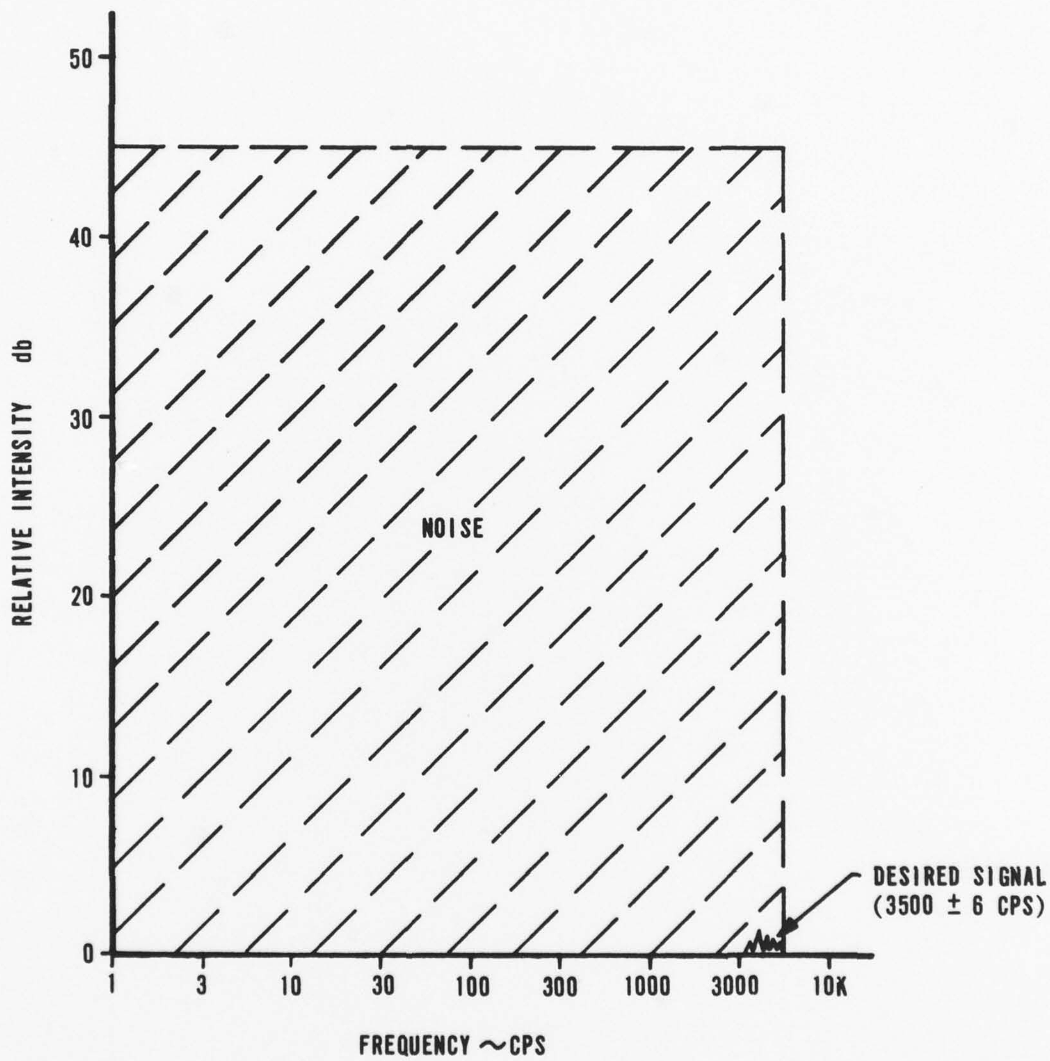

FIGURE 3.1

SPECTRUM OF NOISE AND SCREW SIGNAL

IV. MATCHED FILTER SIGNAL PROCESSING THEORY

As was seen in the previous sections, the frequency spectrum is known but it is very deeply buried in noise. The fundamental purpose of this section is to describe signal processing procedures for extracting the desired signal. It was mentioned that simple band pass filtering of the received waveform increased the signal to noise ratio from -46.2 to -21 db. It will be shown in this section how matched filter techniques for a signal pulse of 0.5 seconds duration can increase the ratio ρ by 36 db. If two dimensional correlation is considered, 60 db can be realized, in which case, the signal can be successfully received. It remains for the next section, dealing with the implementation, to show how 100 nanosecond integrated circuits can produced a near real-time system.

The very words "matched filter" imply that a filter shape is being matched to a signal. Therefore the shape of the received signal masked by noise must be known. In general, the wake can have an average velocity in addition to the screw velocities within the wake which produce the FM effect. The average velocity has the effect of shifting the whole spectrum. It is a formidable problem to design a series of matched filters for a number of screw speeds and to compensate for spectrum shifts. This is why, in this model, attention is paid to the direct signal reflected from the screw and not to the modulation effects it produces on the main submarine hull return. The character of the latter signal is considered to be too dependent upon the physical conditions to be amenable to matched filter techniques. In retrospect, if a prior knowledge about the signal was available, it would be unreasonable to expect such improvements in ρ due to data processing.

The need for a battery of matched filters as proposed in the preceding paragraph may be eliminated, since the Bessel functions that determine the amplitudes of the side bands are in themselves a function of the modulation index. A relationship may be established between the screw rpm and the index and subsequently with the particular echo side band or side bands that peak under certain conditions. It would seem plausible to construct a select number of filters that would detect discrete changes in screw rpm.

In explaining the use of matched filters, there are four things that must be treated: (1) model the sonar problem in terms of a communication problem,(2) show that a linear matched filter operating on the received signal will maximize the signal to noise ratio, (3) show how to calculate the coefficients for this digital filter, and (4) develop a decision statistic that will give a "yes or no answer" as to whether or not the desired screw signature is present. Reference (2) treats the subject in detail in regard to the development of discrete filters for the detection of nuclear explosions.

The sonar problem can be modeled as shown in Figure 4.1.

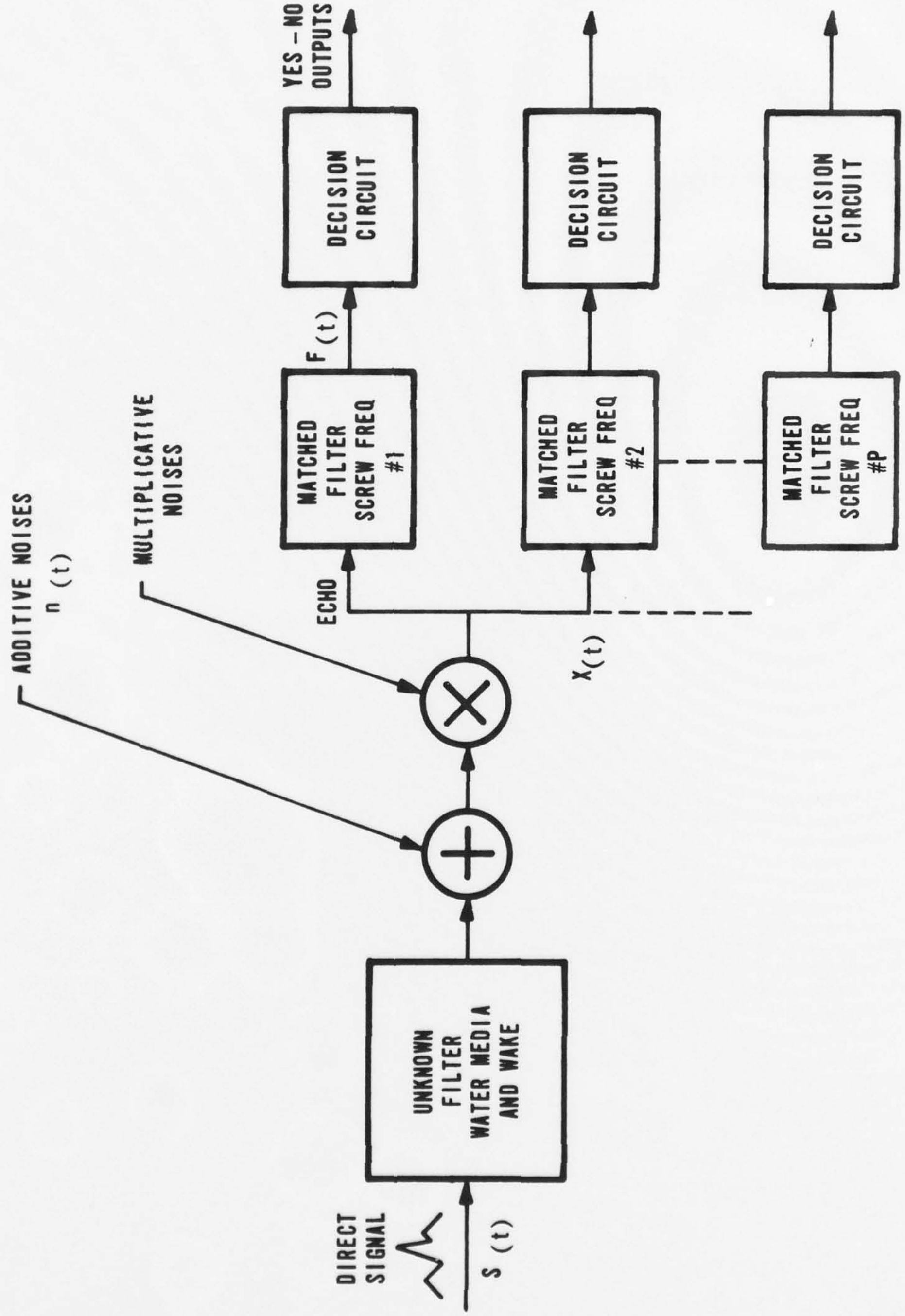

FIGURE 4.1

BLOCK DIAGRAM OF SONAR SYSTEM AND SIGNAL PROCESSING EQUIPMENT

This model parallels the communication problem modeled in Reference (3) except for the fact that multiplicative noises have been included. Continuing with past logic, the direct signal is an a.c. pulse of 0.5 seconds duration which has a Fourier spectrum as shown. The unknown filter has been included for generality and represents the dispersive effects on the signal by the water media and wake. However, since the direct signal is narrow band (3500 $\pm$ 6 cps) the unknown filter effects can be disregarded. Additive noises include such things as reverberation, submarine hull return, and media fluctuations that do not depend upon the presence of the wake return. Little has been said about multiplicative noise, but it represents modulations on the screw echo and is not present in the absence of the other signal. Also, additive noises cause common reinforcements and cancelations between signals while multiplicative noises cause spectrum shifts. In this regard, any relative velocity between ships will be considered a latter type noise. Multiplicative noises will seemingly cause no problem in calculating the coefficients for the digital filter. It is expected that these noises will simply reduce the signal to noise ratio to something less than optimum.

Figure 4.1 shows a number of discrete matched filters each calculated to respond to a different screw rpm signal. The decision circuits have thresholds such that they turn "on" when both screw signal and noise are present. Implementation of this circuitry and estimations as to the computing time are left for Section V.

There are several methods which prove that matched filters produce a maximum signal to noise ratio. The proof to be given here is based on the Schwarz inequality given in Reference (3). This development is not intended to be mathematically rigorous, but merely to show the reader the form of a proof and where reference material can be found.

Referring to Figure 4.1, suppose that the echo signal $X(t)$ is composed of a screw signal $S(t)$ and noise $N(t)$. Moreover $N(t)$ is stationary white noise (constant power with frequency) and has a density of $N_o/2$ watts/cps. If the matched filters are assumed to be linear, then their output of signal plus noise will be larger than for the case of noise alone. This instantaneous power at a time t, when the screw signal is completely in the linear filter

will now be derived.

Let $G(j2\pi f)$ be the transfer function of the filter and $S(j2\pi f)$ be the spectrum of the screw signal. The product of these two quantities will give the spectrum at the filter output. Substitution of this product into the integrand of the familiar Fourier transform will give the variation of power as a function time.

$$Y(t_1) = \int_{-\infty}^{\infty} S(j2\pi f)\, G(j2\pi f)\, e^{j2\pi f t_1}\, df \qquad (1)$$

Since the noise density was assumed stationary, the average power does not change appreciably with time. Multiplication of $N_o/2$ and $/G(j2\pi f)/^2$ will give the noise density spectrum at the filter output, and integration over all frequencies will give the average noise power.

$$\frac{N_o}{2} \int_{-\infty}^{\infty} /G(j2\pi f)/^2\, df \qquad (2)$$

A ratio of the square of the equation (1) to (2) is the power ratio that it is desired to maximize.

$$\rho = \frac{2\left(\int_{-\infty}^{\infty} S(j2\pi f)\, G(j2\pi f)\, e^{j2\pi f t_1}\, df\right)^2}{N_o \int_{-\infty}^{\infty} /\, G(j2\pi f)/^2\, df} \qquad (3)$$

The Schwarz inequality is a general expression and can be written as

$$/\int f(x)\, g(x)\, dx\, /^2 \leq \int /\, f(x)/^2\, dx \int /\, g(x)/^2\, dx \qquad (4)$$

Looking at equations (3) and (4) and letting $f(x) = S(j2\pi f)\, e^{j2\pi f t_1}$ and $g(x) = G(j2\pi f)$, equation (3) can be expressed as an inequality

$$\rho \leq \frac{2}{N_o} \int_{-\infty}^{\infty} /\, S(j2\pi f)/^2\, df \qquad (5)$$

Recognizing that the integral in equation (5) is the total power in the screw signal E, this equation can be finally expressed as

$$\rho \leq \frac{2E}{N_o} \qquad (6)$$

Equation (6) states that ρ is only equal to $2E/N_o$ when $S(j2\pi f)$ and $G(j2\pi f)$ in the numerator of equation (3) are complex conjugates of each other.

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Hence, when the filter transfer function is a complex conjugate of the screw signal spectrum, ρ the signal to noise ratio is a maximum.

great

Note that nothing about the statistics of the noise has been said except also additive that it is stationary and white. If the noise has an arbitrary density spectrum then a similar development gives

$$G(j2\pi f) = \frac{S^*(j2\pi f)}{N(j2\pi f)} \tag{7}$$

Thus, the form of the matched filter may be thought of as composed of two filters: first, a filter with an inverse spectrum of the noise spectrum to "whiten" the noise, and second, the conjugate filter.

It would be of interest now to calculate the gain in signal to noise ratio realized through the use of a matched filter. Equation (6) related this ratio at the filter output.

$$\rho_o = \frac{2E}{N_o} \tag{6}$$

The total noise into the filter N_i can be approximated by $B_n N_o$ where B_n is the noise bandwidth. Likewise, the average signal power into the filter can approximated by E/T where T is the signal duration. Substitution into equation (6) gives

$$\rho_o = 2TB_n \frac{P_{in}}{P_{out}} = 2TB_n \rho_i \tag{7}$$

where

$$\rho_i = \frac{P_{in}}{P_{out}}$$

Hence, signal to noise power ratio at the filter output equals $2TB_n$ times the signal to noise power ratio at the filter input. For the AN/SQS 26 sonar, the db gain is

$$10 \log 2TB_n = (2)x(0.5) \times (4 \times 10^3) = 36 \text{ db} \tag{8}$$

A shorter pulse such as is employed in the AN/SQS 23 sonar would make it impossible to realize this degree of gain.

A filter can be described analytically in the frequency domain by its transfer function and in the time domain by its impulse response. The impulse

response has value because it is the filter output in response to a stylized input. Following the discussion in reference (4), the digitized values of this response out to a point where the response is zero can be represented by a sequence of numbers.

$$A_n \qquad n = 0, 1, 2 \text{ -------- } M \tag{9}$$

It would be desirable at this point to devleop an expression for calculating the filter characteristics or A_n's. Reference (3) does this in minimizing the error between the desired and actual output of a hypothetical filter in the presence of signal plus noise. This procedure results in a linear system of M + 1 equations in the unknowns A_n.

$$\sum_{n=o}^{m} A_n R_x (k-n) = R_{xs}(k) \tag{10}$$

$R_K(k-n)$ is the auto-correlation function of, in this case, the echo signal $\left[X_1, X_2, X_3, \text{-----} X_M \right]$ and $R_{xs}(k)$ is the cross-correlation between the echo signal and the screw signal $\left[S_1, S_2, S_3, \text{-----} S_M \right]$. It is important to point out that the A_n's do not depend upon the values of the S's and X's for a given run, but upon the auto-correlation of the X's and the cross-correlation of the S's and X's. Thus, it is possible that the A_n's calculated for one run will hold for many subsequent runs, depending upon the stationarity of the noise and target geometry. It is a guess that the condition of stationarity may apply over two or three minutes. It will be the work of the ultra-high speed, special circuitry to calculate the new auto and cross-correlations at the beginning of the new period. These values can then be applied to equation (10) in a digital general purpose computer to solve for the new filter coefficients. Any "pre-whitening" of the filter can be accomplished at this stage in the time domain, using previous filter information or a prior knowledge about the physics of the problem.

Once the filter coefficients are known, they can be applied in a digital computer to a incoming echo signal to give the filtered output F(t).

$$F(t) = \sum_{n=o}^{m} A_n X(t-nh) \tag{11}$$

where h is the digitizing increment. The final step would be to affect any inter-pulse averaging that might be necessary.

In setting the thresholds on the decision circuits, it would be of interest to examine the mean-square values of the filter outputs in the presence of noise alone and signal plus noise. The mean square output for the case where the noise is known only to be stationary is defined as

$$E \left\{ (X A_n)^2 \right\} \tag{12}$$

E here denotes the time average of the squared matrix multiplication. For the case of noise above, Reference (2) gives the mean-square output.

$$E \left\{ (X A_n)^2 \right\} = A_n^T \phi A_n \tag{13}$$

where ϕ is the correlation matrix of the noise, and A_n^T is the matrix transpose of A_n. Under the conditions of signal plus noise, this output is

$$E \left\{ (X A_n)^2 \right\} = A_n^T S^T S A_n + A_n^T \phi A_n \tag{14}$$

At the time that the screw signal is completely in the filter, the instantaneous power output is equal to the noise power output plus the other term involving S and S^T.

This completes the theory on matched filters. The next section deals with its implementation.

V. IMPLEMENTATION

This step should logically consist of two parts: (1) for the near future, runs with actual data on the IBM 7094 digital computer, and (2) for the more distant future, construction of ultra high speed auto and cross-correlation digital circuitry to check feasibility of a near real-time signal processor.

Experimentation on a general purpose computer permits flexibility in changing design parameters such as filter coefficients to arrive at an optimum system. The exact configuration of the filter will be a function of theory and actual noise characteristics. For instance, reverberation caused by the bounded water media may produce particular frequencies that must be "whitened". It would be important to know how well previous information can be used to compute the new filter coefficients.

This study would be of no practical importance unless a near real-time system could be constructed ultimately. Ultra high speed digital circuitry is now becoming available that makes this goal possible. These integrated circuits are capable of producing a multiplication, add to register, and shift in 0.5 microseconds. These speeds would immeasurably help in computing the auto and cross-correlations discussed in the previous section. Some of the other computing operations relegated in this report to a general purpose computer may also be substantially speeded up by the use of special modules. In summary, the theoretical analysis and the practical implementation might be conducted concurrently to best advantage.

VI. REFERENCES

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